Examples Based on Binomial Probability Distribution

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Binomial Distribution

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- **Final answer:**

$$P(X \geq 4) = 0.014650 + 0.000977 = \boxed{0.015627}$$

Example 2: Sum of Two Binomial Random Variables

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$$X \sim B(4, 0.7), \quad Y \sim B(3, 0.7)$$

Find $P(X + Y \leq 1)$.

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- **Final answer:**

$$P(X + Y \leq 1) = 0.0002187 + 0.0035721 = \boxed{0.0037908}$$

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- **Final answer:**

$$P(X \geq 2) = 1 - \left(\frac{243 + 405}{1024}\right) = 1 - \frac{648}{1024} = \frac{376}{1024} = \frac{47}{128}$$

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- **Final Answer:**

$$\boxed{P(\text{dacoit is still alive}) = 0.0041}$$

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