Solving First Order and First Degree Differential Equations

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- **Solve these simultaneous equations to find h and k.**

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- **If the constants** a, b, c, a', b', c' **do not allow such proportional simplification and the determinant vanishes, the method fails.**
- **In that case, an alternative method (like an integrating factor or substitution based on symmetry) may be needed.**

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- **Let** $Y = vX$ **so that:**

$$\frac{dY}{dX} = v + X\frac{dv}{dX} \Rightarrow v + X\frac{dv}{dX} = \frac{1 - 4v}{1 + v}$$

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$$\int \frac{1+v}{1-5v-v^2}\,dv = \int \frac{1}{X}\,dX$$

- **Integrate the right-hand side:**

$$\int \frac{1}{X}\,dX = \ln|X| + c$$

Solving the Integral and Final Answer

- **The left-hand side integral is solved using the identity:**

$$\int \frac{1+v}{1-5v-v^2}\,dv = \frac{1}{2}\ln|1-5v-v^2| + \tan^{-1}\left(\frac{2v+5}{\sqrt{D}}\right)$$

where D is the discriminant. But for this case, directly:

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- **Final answer:**

$$\frac{1}{2}\ln\left[(x+2)^2 + (y+3)^2\right] + \tan^{-1}\left(\frac{y+3}{x+2}\right) = c$$

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- **Rewrite in the form:**

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- **Rearranged:**

$$\frac{1}{2} \cdot \frac{dv}{dx} = \frac{v + 5}{2v + 5} \Rightarrow \frac{dv}{dx} = \frac{2(v + 5)}{2v + 5}$$

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- **Substitute back $v = 2x + 3y$:**

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