Binomial Distribution: Mean and Variance

Problem Statement

Given

Is it possible a binomial distribution has a mean of 3 and a variance of 4?

Solution

- For a binomial distribution, the mean is given by $\mu = np$.

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- Dividing variance by mean:

$$\frac{npq}{np} = \frac{4}{3} \Rightarrow q = \frac{4}{3}$$

- But q, being a probability, cannot be greater than 1.
- Therefore, the given statement is not possible for a binomial distribution.

Solution

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- Multiply by 10: $100p - 50p^2 = 48$

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- $50p^2 - 100p + 48 = 0$

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- $50p^2 - 100p + 48 = 0$
- Divide by 2: $25p^2 - 50p + 24 = 0$

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- Multiply by 10: $100p - 50p^2 = 48$
- $50p^2 - 100p + 48 = 0$
- Divide by 2: $25p^2 - 50p + 24 = 0$
- Solving, we get $p = \frac{4}{5}$ and $q = \frac{1}{5}$(other root rejected $p > 1$)

Binomial Distribution

The binomial distribution is:

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- $P(X = 4) = \frac{1280}{3125}$

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- $P(X = 4) = \frac{1280}{3125}$
- $P(X = 5) = \frac{1024}{3125}$

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- (iii) Kurtosis

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- Thus, $p = 1 - q = 1 - \frac{5}{6} = \frac{1}{6}$

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- Standard Deviation $= \sqrt{npq} = 5 \Rightarrow npq = 25$
- Divide: $\frac{npq}{np} = \frac{25}{30} \Rightarrow q = \frac{5}{6}$
- Thus, $p = 1 - q = 1 - \frac{5}{6} = \frac{1}{6}$
- $np = 30 \Rightarrow n \times \frac{1}{6} = 30 \Rightarrow n = 180$

Solution: Part (ii) - Skewness

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- $\gamma_1 = \frac{\frac{5}{6}-\frac{1}{6}}{5} = \frac{4/6}{5} = \frac{2}{15}$

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- Simplify: $\beta_2 = 3 + \frac{1-\frac{5}{6}}{25} = 3 + \frac{1}{150}$

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- Thus, $\gamma_2 = \beta_2 - 3 = \frac{1}{150} > 0$

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- Simplify: $\beta_2 = 3 + \frac{1-\frac{5}{6}}{25} = 3 + \frac{1}{150}$
- Thus, $\gamma_2 = \beta_2 - 3 = \frac{1}{150} > 0$
- The distribution is **leptokurtic**.

Problem Statement

Number of Heads	Frequencies
0	7
1	6
2	19
3	35
4	30
5	23
6	7
7	1

- Seven coins are tossed and the number of heads is recorded.

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- Seven coins are tossed and the number of heads is recorded.
- The experiment is repeated 128 times.
- The following frequency distribution is obtained:
- Fit a binomial distribution assuming the coin is unbiased.

Solution: Step 1

- Given: The coin is unbiased, so $p = \frac{1}{2}$, $q = \frac{1}{2}$

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- Number of trials $n = 7$, Number of experiments $N = 128$
- The binomial probability is:

$$p(x) = \binom{7}{x} \left(\frac{1}{2}\right)^{x} \left(\frac{1}{2}\right)^{7-x}$$

Solution: Step 1

- Given: The coin is unbiased, so $p = \frac{1}{2}$, $q = \frac{1}{2}$
- Number of trials $n = 7$, Number of experiments $N = 128$
- The binomial probability is:

$$p(x) = \binom{7}{x} \left(\frac{1}{2}\right)^{x} \left(\frac{1}{2}\right)^{7-x}$$

- Expected frequency: $f(x) = N \times p(x)$

Solution: Step 2 - Expected Frequencies

- Calculate each probability:

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  - $p(0) = \binom{7}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^7 = \frac{1}{128}$

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  - $p(0) = \binom{7}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^7 = \frac{1}{128}$
  - $p(1) = \binom{7}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^6 = \frac{7}{128}$

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  - $p(2) = \binom{7}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^5 = \frac{21}{128}$

Solution: Step 2 - Expected Frequencies

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  - $p(1) = \binom{7}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^6 = \frac{7}{128}$
  - $p(2) = \binom{7}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^5 = \frac{21}{128}$
  - $p(3) = \binom{7}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^4 = \frac{35}{128}$

Solution: Step 2 - Expected Frequencies (Contd.)

- Continuing:

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  - $p(4) = \frac{35}{128}$

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- Continuing:
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  - $p(5) = \frac{21}{128}$

Solution: Step 2 - Expected Frequencies (Contd.)

- Continuing:
  - $p(4) = \frac{35}{128}$
  - $p(5) = \frac{21}{128}$
  - $p(6) = \frac{7}{128}$

Solution: Step 2 - Expected Frequencies (Contd.)

- Continuing:
  - $p(4) = \frac{35}{128}$
  - $p(5) = \frac{21}{128}$
  - $p(6) = \frac{7}{128}$
  - $p(7) = \frac{1}{128}$

Solution: Step 2 - Expected Frequencies (Contd.)

- Continuing:
    - $p(4) = \frac{35}{128}$
    - $p(5) = \frac{21}{128}$
    - $p(6) = \frac{7}{128}$
    - $p(7) = \frac{1}{128}$
- Expected Frequencies:
    - $f(0) = 128 \times \frac{1}{128} = 1$
    - $f(1) = 128 \times \frac{7}{128} = 7$
    - $f(2) = 128 \times \frac{21}{128} = 21$
    - $f(3) = 128 \times \frac{35}{128} = 35$
    - $f(4) = 35,\ f(5) = 21,\ f(6) = 7,\ f(7) = 1$

Summary Table

Number of Heads (X)	Observed Frequencies	Expected Frequencies
0	7	1
1	6	7
2	19	21
3	35	35
4	30	35
5	23	21
6	7	7
7	1	1

Problem Statement

Out of 800 families with 4 children each, how many families would you expect to have 3 boys and 1 girl, assuming equal probability of boys and girls?

Solution: Step 1

- Given:

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  - Number of children $n = 4$
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- Given:
  - Number of families $N = 800$
  - Number of children $n = 4$
  - Probability of boy $p = \frac{1}{2}$
  - Probability of girl $q = \frac{1}{2}$

- We need the probability of exactly 3 boys and 1 girl:

$$P(X = 3) = \binom{4}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1$$

Solution: Step 2

- We need the probability of exactly 3 boys and 1 girl:

$$P(X = 3) = \binom{4}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1$$

- Calculate:

$$P(X = 3) = 4 \times \frac{1}{8} \times \frac{1}{2} = 4 \times \frac{1}{16} = \frac{1}{4}$$

Solution: Final Step

- Expected number of families:

$$E = N \times P(X = 3) = 800 \times \frac{1}{4} = 200$$

Solution: Final Step

- Expected number of families:

$$E = N \times P(X = 3) = 800 \times \frac{1}{4} = 200$$

- **Therefore, expected number of families with 3 boys and 1 girl is 200.**

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Thank You!