Understand Poisson Distribution

Bindeshwar Singh Kushwaha

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- Examples:
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    - Number of defects in production per item.
    - Number of accidents during a time interval.

Poisson Distribution (contd.)

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- Poisson is a limiting case of binomial distribution under:
  - $n \to \infty$
  - $p \to 0$
  - $np = \lambda$, a finite constant

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- $e \approx 2.7183$, base of natural logarithm.

Moments of Poisson Distribution

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- Simplifying:

$$\begin{aligned}
\mu'_1 &= e^{-\lambda} \sum_{x=1}^{\infty} \frac{x\lambda^x}{x!} \\
&= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda \cdot \lambda^{x-1}}{(x-1)!} \\
&= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} \\
&= \lambda e^{-\lambda} \cdot e^{\lambda} = \lambda
\end{aligned}$$

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- $\therefore$ Mean $= \lambda$

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- Split into two sums:

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- Change index for first term ($x \geq 2$):

$$= \sum_{x=2}^{\infty} x(x-1) \cdot \frac{e^{-\lambda}\lambda^x}{x!} + \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

Second Moment μ_2' of Poisson Distribution

- Factor common terms:

$$= e^{-\lambda}\left[\sum_{x=2}^{\infty}\frac{\lambda^x}{(x-2)!}+\sum_{x=0}^{\infty}\frac{x\lambda^x}{x!}\right]$$

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$$= e^{-\lambda}\left[\lambda^2\sum_{x=0}^{\infty}\frac{\lambda^x}{x!}+\lambda\sum_{x=0}^{\infty}\frac{\lambda^x}{x!}\right]$$

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- Use identity: $\sum_{x=0}^{\infty}\frac{\lambda^x}{x!} = e^{\lambda}$

$$= e^{-\lambda}\cdot(\lambda^2+\lambda)\cdot e^{\lambda} = \lambda^2+\lambda$$

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- ∴ Second raw moment:

$$\boxed{\mu_2' = \lambda^2+\lambda}$$

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- $\therefore$ For Poisson Distribution:

$$\boxed{\text{Mean} = \lambda, \quad \text{Variance} = \lambda}$$

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- Expanding:

$$\mu_3' = \sum_{x=0}^{\infty} [x(x-1)(x-2) + 3x(x-1) + x] \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$
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- Where:

$$A = \sum_{x=0}^{\infty} x(x-1)(x-2) \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

$$B = 3\sum_{x=0}^{\infty} x(x-1) \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

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Third Moment μ'_3 of Poisson Distribution: Simplification

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- Final Result:

$$\mu_3' = A + B + C = \lambda^3 + 3\lambda^2 + \lambda$$

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- Final Result:

$$\mu_3' = A + B + C = \lambda^3 + 3\lambda^2 + \lambda$$

- Third order central moment:

$$\mu_3 = \mu_3' - 3\mu_2'\mu + 2(\mu_1')^3 = \lambda$$

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$$C = 7\sum_{x=0}^{\infty} x(x-1) \cdot \frac{e^{-\lambda}\lambda^x}{x!}, \quad D = \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

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- Simplifying C and D:

$$C = 7e^{-\lambda}\lambda^2 \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} = 7\lambda^2, \quad D = \lambda$$

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- Final Result:

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- Final Result:

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- Fourth central moment:

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4 = 3\lambda^2 + \lambda$$

Skewness and Kurtosis of Poisson Distribution

- Skewness is given by:

$$\text{Skewness} = \gamma_1 = \frac{\mu_3}{\sigma^3} = \frac{\lambda}{\lambda^{3/2}} = \frac{1}{\sqrt{\lambda}}$$

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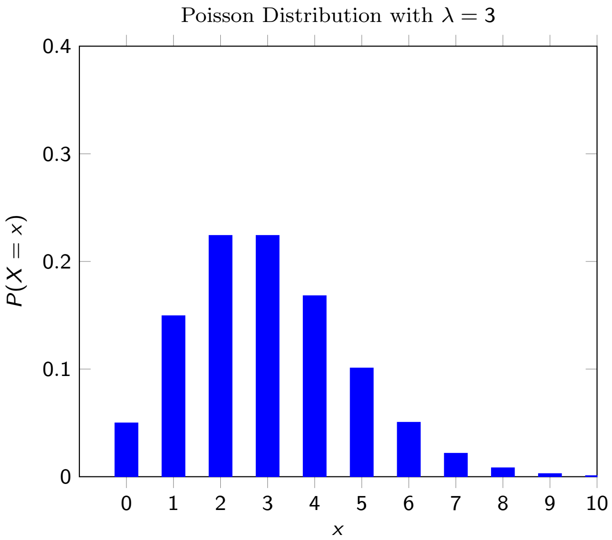
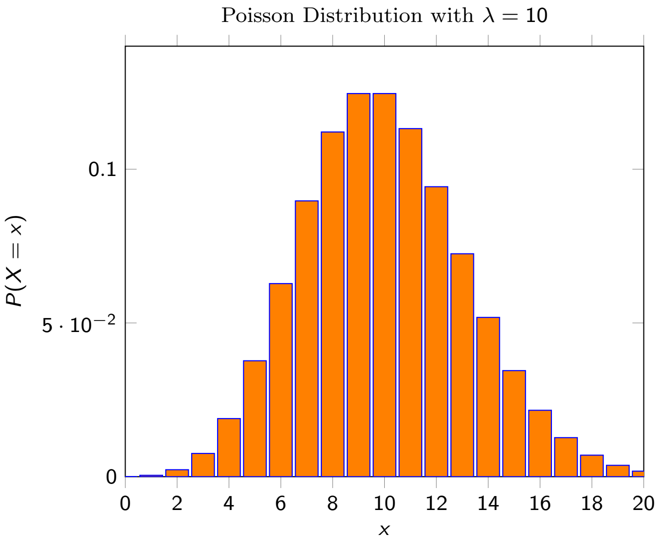
  - Poisson distribution is positively skewed.
  - It is leptokurtic when λ is small.

Poisson Distribution Plot



Poisson Distribution: $\lambda = 10$



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Thank You!