Span and Intersection of Subspaces

Bindeshwar Singh Kushwaha

PostNetwork Academy

Linear Span: Definition and Properties

- **Given vectors $u_1, u_2, \ldots, u_m$ in a vector space V, their linear span is the set of all linear combinations:**

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- **The span of a set of vectors is a subspace of V.**
- **Theorem:**
  - **(i) span(S) is a subspace of V that contains S.**
  - **(ii) If W is a subspace of V containing S, then span(S) $\subseteq W$.**

Numerical Examples of Linear Spans

- **Example 1: Span of $u_1 = (1,0)$ and $u_2 = (0,1)$ in $\mathbb{R}^2$ is the entire plane $\mathbb{R}^2$.**

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- **Example 4: Span of** $u_1 = (1, 1, 1)$**,** $u_2 = (0, 1, 2)$**,** $u_3 = (1, 0, 1)$ **in** $\mathbb{R}^3$ **is the entire space** $\mathbb{R}^3$**.**

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- **Example 3: Span of $u_1 = (1, 0, 0)$, $u_2 = (0, 1, 0)$ in $\mathbb{R}^3$ is the xy-plane.**
- **Example 4: Span of $u_1 = (1, 1, 1)$, $u_2 = (0, 1, 2)$, $u_3 = (1, 0, 1)$ in $\mathbb{R}^3$ is the entire space $\mathbb{R}^3$.**
- **Example 5: Span of the rows of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ is the line defined by $(1, 2)$ because the second row is a multiple of the first.**

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- **Let $u, v \in U \cap W$, so $u, v \in U$ and $u, v \in W$.**

W

v

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- **Let $u, v \in U \cap W$, so $u, v \in U$ and $u, v \in W$.**
- **Since U and W are subspaces, $au + bv \in U$ and $au + bv \in W$ for all scalars $a, b \in K$.**

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- **Therefore, $au + bv \in U \cap W$.**

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- **Let U and W be subspaces of a vector space V.**
- **The zero vector belongs to both U and W, so $0 \in U \cap W$.**
- **Let $u, v \in U \cap W$, so $u, v \in U$ and $u, v \in W$.**
- **Since U and W are subspaces, $au + bv \in U$ and $au + bv \in W$ for all scalars $a, b \in K$.**
- **Therefore, $au + bv \in U \cap W$.**
- **Hence, $U \cap W$ is a subspace of V.**

Theorem

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The intersection of any number of subspaces of a vector space V is a subspace of V.

Numerical Example 1

- **In** $\mathbb{R}^2$**:** $U = \mathbf{span}\{(1,0)\}$**,** $W = \mathbf{span}\{(0,1)\}$

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- **Intersection:** $U \cap W = \{(0,0)\}$

Numerical Example 2

- **In** $\mathbb{R}^3$**:** $U = \textbf{span}\{(1,0,0),(0,1,0)\}$

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- $W = \textbf{span}\{(1,1,0)\}$
- **Intersection:** $U \cap W = \textbf{span}\{(1,1,0)\}$

Numerical Example 3

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- $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
- **Let's assume B lies in the column space of A, so the intersection is: span$\{(1,2,3)^T\}$**

Numerical Example 4

- $U = \textbf{span}\{(1,2,3),(4,5,6)\}$

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- $U = \mathbf{span}\{(1,2,3),(4,5,6)\}$
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- $U = \textbf{span}\{(1, 2, 3), (4, 5, 6)\}$
- $W = \textbf{span}\{(1, 0, 0), (0, 1, 0)\}$
- **The intersection lies in the XY-plane; the exact intersection can be found by solving the system.**

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