Fitting Binomial Distribution

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- Fitting a binomial distribution involves comparing observed frequencies with expected frequencies derived from the binomial probability formula.
- The recurrence relation simplifies the process of finding probabilities.
- This technique is useful for testing if a dataset follows a binomial distribution.

Binomial Probability Function

- The binomial probability function is:

$$p(x) = {}^{n}C_{x} p^{x} q^{n-x}$$

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$$p(x) = {}^{n}C_{x} p^{x} q^{n-x}$$

- Where:
  - n = number of trials
  - p = probability of success
  - $q = 1 - p$ = probability of failure

Derivation of Recurrence Relation

- Start with the binomial probability function:

$$p(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

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- Divide $p(x+1)$ by $p(x)$:

$$\frac{p(x+1)}{p(x)} = \frac{(n-x)}{(x+1)} \cdot \frac{p}{q}$$

- Therefore, the recurrence relation is:

$$p(x+1) = \frac{(n-x)}{(x+1)} \cdot \frac{p}{q} \cdot p(x)$$

Example: Tossing 4 Coins

- Observed data:
  - 0 heads: 15 times
  - 1 head: 35 times
  - 2 heads: 90 times
  - 3 heads: 40 times
  - 4 heads: 20 times

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- Observed data:
  - 0 heads: 15 times
  - 1 head: 35 times
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- Total number of tosses: 200

Calculation of Mean and Probability

- Mean:

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Calculation of Mean and Probability

- Mean:

$$\bar{x} = \frac{415}{200} = 2.075$$

- $p = \frac{2.075}{4} = 0.5188, \qquad q = 1 - p = 0.4812$
- Calculate:

$$p(0) = q^4 = (0.4812)^4 = 0.0536$$

Using Recurrence Relation

- Use:

$$p(x+1) = \frac{(n-x)}{(x+1)} \cdot \frac{p}{q} \cdot p(x)$$

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- Calculate:
  - $p(1) = 4 \times \frac{0.5188}{0.4812} \times 0.0536 = 0.23115$

Using Recurrence Relation

- Use:

$$p(x+1) = \frac{(n-x)}{(x+1)} \cdot \frac{p}{q} \cdot p(x)$$

- Calculate:
  - $p(1) = 4 \times \frac{0.5188}{0.4812} \times 0.0536 = 0.23115$
  - $p(2) = \frac{3}{2} \times \frac{0.5188}{0.4812} \times 0.23115 = 0.37382$

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$$p(x+1) = \frac{(n-x)}{(x+1)} \cdot \frac{p}{q} \cdot p(x)$$

- Calculate:
  - $p(1) = 4 \times \frac{0.5188}{0.4812} \times 0.0536 = 0.23115$
  - $p(2) = \frac{3}{2} \times \frac{0.5188}{0.4812} \times 0.23115 = 0.37382$
  - $p(3) = 2 \times \frac{0.5188}{0.4812} \times 0.37382 = 0.26859$

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$$p(x+1) = \frac{(n-x)}{(x+1)} \cdot \frac{p}{q} \cdot p(x)$$

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  - $p(3) = 2 \times \frac{0.5188}{0.4812} \times 0.37382 = 0.26859$
  - $p(4) = \frac{1}{4} \times \frac{0.5188}{0.4812} \times 0.26859 = 0.0724$

Detailed Summary Table

Number of Heads (X)	Recurrence Relation	p(x)	Expected Frequency
0	Initial: q^4	0.0536	10.72
1	$4 \times \frac{p}{q} \times p(0)$	0.23115	46.23
2	$\frac{3}{2} \times \frac{p}{q} \times p(1)$	0.37382	74.76
3	$2 \times \frac{p}{q} \times p(2)$	0.26859	53.71
4	$\frac{1}{4} \times \frac{p}{q} \times p(3)$	0.0724	14.48

Conclusion

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- The expected frequencies using the binomial model are close to the observed frequencies.
- The example supports the assumption that the coin is unbiased.
- The recurrence relation simplifies stepwise calculation of probabilities.

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