Moment Generating Function of Binomial Distribution

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Definition of Moment Generating Function

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- **For a discrete random variable:**

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Series Expansion of MGF

- **Using Taylor series expansion of e^{tX}:**

$$e^{tX} = 1 + tX + \frac{t^2X^2}{2!} + \frac{t^3X^3}{3!} + \cdots + \frac{t^rX^r}{r!} + \cdots$$

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- **This is:**

$$M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!}\mu'_r$$

where $\mu'_r = E(X^r)$ is the rth moment about origin.

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- **Example:**

$$\frac{d}{dt} M_X(t) = E\left[Xe^{tX}\right], \quad \textbf{and at } t = 0, \quad E(X)$$

MGF of Binomial Distribution

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- **Mean (μ):**

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- **Variance (σ^2):**

$$\sigma^2 = M''_X(0) - (M'_X(0))^2 = npq$$

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