

Poisson Distribution Numerical Examples

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Example-1: Poisson Distribution Problem

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 - (b) At least two trucks arrive

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- The Poisson probability mass function is given by:

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(a) Probability of No Heavy Truck

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- Numerically,

$$P(X = 0) \approx 0.1353$$

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- **Note:** We often use $P(X \geq a) = 1 - P(X < a)$ in Poisson problems.

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- Let X be the Poisson variate representing the number of individuals suffering from a bad reaction.

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- Poisson distribution:

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, \dots$$

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- So,

$$P(X = 3) = \frac{e^{-1.5} \cdot 3.375}{6} \approx 0.1255$$

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- Approximate:

$$P(X > 2) = 1 - e^{-1.5} \cdot 3.625 \approx 1 - 0.2231 \cdot 3.625 = 1 - 0.8097 = 0.1903$$

Example-3: Moments of Poisson Distribution

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- Variance of Poisson: $\lambda = 1.44$
- Central moment of order 3: $\mu_3 = \lambda = 1.44$
- Central moment of order 4:

$$\mu_4 = 3\lambda^2 + \lambda = 3(1.44)^2 + 1.44 = 7.66$$

Example-4: Solve for λ from Probability Condition

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Find: The mean and variance of the distribution.

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- Cancel $e^{-\lambda}$, multiply both sides:

$$\frac{\lambda}{1} = 2 \cdot \frac{\lambda^2}{2} \Rightarrow \lambda = \lambda^2 \Rightarrow \lambda(\lambda - 1) = 0$$

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- So,

$$\text{Mean} = 1, \quad \text{Variance} = 1$$

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- If $X \sim \text{Poisson}(1)$ and $Y \sim \text{Poisson}(2)$ are independent,

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- The sum $X + Y \sim \text{Poisson}(\lambda = 1 + 2 = 3)$
- Let $W = X + Y$, then:

$$P(W = w) = \frac{e^{-3} \cdot 3^w}{w!}, \quad w = 0, 1, 2, \dots$$

Example-5: Final Calculation

Required Probability: $P(X + Y < 2) = P(W < 2)$

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- Compute each:

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- Using $e^{-3} \approx 0.0498$, we get:

$$P(W < 2) = 0.0498 + 3 \cdot 0.0498 = 0.0498(1 + 3) = 0.0498 \cdot 4 = 0.1992$$

Example-6

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- In a particular mine, there are 350 miners.
- What is the probability that at least one fatal accident will occur in a year?

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- Since n is large and p is small, we use the Poisson approximation:

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- Hence,

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- **Final Answer:** There is a **22.12%** chance of at least one fatal accident in a year.

Example-7:Poisson Distribution Validity Check

Q: The mean and standard deviation of a Poisson distribution are 6 and 2 respectively. Test the validity of this statement.

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- But then $\lambda = 4$ from the variance.
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- **Conclusion:** The statement is **invalid**.

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- Hence, find $P(X = 0)$ and $P(X = 4)$.

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- Solving: $\lambda(\lambda - 2) = 0 \Rightarrow \lambda = 0$ or 2

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- Reject $\lambda = 0$, so $\lambda = 2$

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$$= \frac{2}{3} \cdot 0.1353 = 0.0902$$

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Thank You!