Fitting of Poisson Distribution

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- **We find the theoretical (expected) frequencies corresponding to each value of the Poisson variate.**
- **Using recurrence relation makes finding probabilities easier.**
- **We first establish the recurrence relation for probabilities and then define the Poisson frequency distribution.**

Recurrence Formula for the Probabilities of Poisson Distribution

For a Poisson distribution with parameter λ, we have:

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Dividing (2) by (1), we obtain:

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$$\Rightarrow \quad p(x+1) = \frac{\lambda}{x+1}\, p(x) \quad \ldots(3)$$

Using the Recurrence Relation

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- We can compute $p(1), p(2), p(3), \ldots$ successively.

Poisson Frequency Distribution

If an experiment satisfies the requirements of Poisson distribution and is repeated N times:

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- **Substituting the Poisson probability:**

$$f(x) = N \cdot \frac{e^{-\lambda}\lambda^{x}}{x!}$$

- **Hence,**

$$f(x) = N \cdot \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0, 1, 2, \ldots$$

Example: Defective Bottles Problem

A manufacturer produces medicine bottles, 0.1% are defective. Bottles are packed in boxes of 500. A drug manufacturer buys 100 boxes. Find how many boxes will contain at least two defective bottles.

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$$p = \frac{0.1}{100} = 0.001$$

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- **Number of bottles in a box = 500**

$$\lambda = np = 500 \times 0.001 = 0.5$$

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$$p = \frac{0.1}{100} = 0.001$$

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- **Number of boxes:**

$$N = 100$$

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$$p = \frac{0.1}{100} = 0.001$$

- **Number of bottles in a box = 500**

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- **Number of boxes:**

$$N = 100$$

- **Poisson probability:**

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-0.5}(0.5)^x}{x!}, \quad x = 0, 1, 2, \ldots$$

Step 2: Probability of At Least 2 Defective Bottles

- **We need:**

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$$= 1 - \left[\frac{e^{-0.5}(0.5)^0}{0!} + \frac{e^{-0.5}(0.5)^1}{1!}\right]$$

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- **Which means:**
$$P(X \geq 2) = 1 - \left[P(X = 0) + P(X = 1)\right]$$

- **Substituting values:**
$$= 1 - \left[\frac{e^{-0.5}(0.5)^0}{0!} + \frac{e^{-0.5}(0.5)^1}{1!}\right]$$

- **Simplify:**
$$= 1 - e^{-0.5}[1 + 0.5] = 1 - (0.6065)(1.5) = 1 - 0.90975 = 0.09025$$

Step 3: Expected Number of Boxes

- **Expected number of boxes with at least two defective bottles:**

$$= N \times P(X \geq 2)$$

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- **Substituting:**

$$= 100 \times 0.09025 = 9.025$$

- $\therefore$ **About 9 boxes will contain at least 2 defective bottles.**

Process of Fitting a Poisson Distribution

For fitting a Poisson distribution to the observed data, you are to proceed as described in the following steps:

- **First we obtain mean of the given distribution i.e.**

$$\bar{x} = \frac{\sum fx}{\sum f}$$

being mean, take this as the value of λ.

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For fitting a Poisson distribution to the observed data, you are to proceed as described in the following steps:

- **First we obtain mean of the given distribution i.e.**

$$\bar{x} = \frac{\sum fx}{\sum f}$$

  being mean, take this as the value of λ.

- **Next we obtain**

$$p(0) = e^{-\lambda}$$

Process of Fitting a Poisson Distribution

- The recurrence relation

$$p(x+1) = \frac{\lambda}{x+1} p(x)$$

is then used to compute the values of $p(1), p(2), p(3), \ldots$

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- The recurrence relation
$$p(x+1) = \frac{\lambda}{x+1} p(x)$$
is then used to compute the values of $p(1), p(2), p(3), \ldots$
- The probabilities obtained in the preceding two steps are then multiplied with N to get expected/theoretical frequencies i.e.
$$f(x) = N \cdot P[X = x], \quad x = 0, 1, 2, \ldots$$

Problem Statement

Data on Aircraft Accidents

The following data gives the frequencies of aircraft accidents experienced by 2480 pilots during a certain period.

Number of Accidents (X)	0	1	2	3	4	5
Frequencies (f)	1970	422	71	13	3	1

Step 1: Calculate the Mean (λ)

- **Total Number of Pilots (N) is the sum of frequencies:**

$$N = \sum f = 1970 + 422 + 71 + 13 + 3 + 1 = 2480$$

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- **Total Number of Accidents ($\sum fX$):**
  - $0(1970) + 1(422) + 2(71) + 3(13) + 4(3) + 5(1)$
  - $= 0 + 422 + 142 + 39 + 12 + 5 = 620$

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  - $0(1970) + 1(422) + 2(71) + 3(13) + 4(3) + 5(1)$
  - $= 0 + 422 + 142 + 39 + 12 + 5 = 620$
- **Mean (λ) is the average number of accidents:**

$$\lambda = \frac{\sum fX}{\sum f} = \frac{620}{2480}$$

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- **Total Number of Pilots (N) is the sum of frequencies:**

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- **Total Number of Accidents ($\sum fX$):**
  - $0(1970) + 1(422) + 2(71) + 3(13) + 4(3) + 5(1)$
  - $= 0 + 422 + 142 + 39 + 12 + 5 = 620$
- **Mean (λ) is the average number of accidents:**

$$\lambda = \frac{\sum fX}{\sum f} = \frac{620}{2480}$$

- **Result:**

$$\lambda = 0.25$$

Step 2: Calculate Initial Probability $p(0)$

- **The Poisson Probability Mass Function (PMF) is:**

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

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- **For $x = 0$, the probability $p(0)$ is:**

$$p(0) = P(X = 0) = \frac{e^{-0.25}(0.25)^{0}}{0!} = e^{-0.25}$$

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- **Value from table (or calculator, as per the source):**

$$e^{-0.25} \approx 0.7788$$

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- **Value from table (or calculator, as per the source):**

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- **Result:**

$$p(0) = 0.7788$$

Step 3: Calculate Probabilities $p(x)$ using Recurrence

- **Poisson Recurrence Relation:**

$$p(x+1) = p(x) \times \frac{\lambda}{x+1}$$

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- **Applying the relation for $\lambda = 0.25$:**
  - $p(1)$: $p(0) \times \frac{0.25}{0+1} = 0.7788 \times 0.25 = 0.1947$

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  - $p(3)$: $p(2) \times \frac{0.25}{2+1} \approx 0.0243 \times 0.0833 \approx 0.0020$
  - $p(4)$: $p(3) \times \frac{0.25}{3+1} \approx 0.0020 \times 0.0625 \approx 0.0001$

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  - $p(4)$: $p(3) \times \frac{0.25}{3+1} \approx 0.0020 \times 0.0625 \approx 0.0001$
  - $p(5)$: $p(4) \times \frac{0.25}{4+1} \approx 0.0001 \times 0.05 \approx 0.000005$

Step 4: Calculate Theoretical Frequencies

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  - $f(3)$: $2480 \times 0.0020 \approx 4.96 \rightarrow 5$
  - $f(4)$: $2480 \times 0.0001 \approx 0.248 \rightarrow 0$
  - $f(5)$: $2480 \times 0.000005 \approx 0 \rightarrow 0$

Conclusion: Comparison of Frequencies

Summary Table

The Poisson distribution with $\lambda = 0.25$ provides a good fit for the observed data, as the theoretical frequencies closely match the actual observed frequencies.

Accidents (X)	0	1	2	3	4	5
Observed (f)	1970	422	71	13	3	1
Theoretical ($f(x)$)	1931	483	60	5	0	0

Example : Problem Statement

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- **Total Consignment (N): The consignment contains 20000 packets.**

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- Probability of a Defective Pen (p):

$$p = \frac{1}{500}$$

- Packet Size (n): The pens are supplied in packets of 10.
- Total Consignment (N): The consignment contains 20000 packets.
- Goal: Calculate the approximate number of packets containing:
  1. One defective pen ($X = 1$).
  2. Two defective pens ($X = 2$).

Step 1: Calculate the Mean (λ)

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$$\lambda = np$$

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- Calculation:

$$\lambda = 10 \times \frac{1}{500} = \frac{10}{500}$$

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- Calculation:

$$\lambda = 10 \times \frac{1}{500} = \frac{10}{500}$$

- Result:

$$\lambda = 0.02$$

Step 2: The Poisson Probability Formula

- **The probability of exactly x defective pens in a packet, $P[X = x]$, is given by the Poisson PMF:**

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- The Expected/Theoretical Frequency $f(x)$ for a consignment of $N = 20000$ packets is:

$$f(x) = N \cdot P[X = x] = 20000 \cdot \frac{e^{-0.02}(0.02)^{x}}{x!}$$

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$$f(x) = N \cdot P[X = x] = 20000 \cdot \frac{e^{-0.02}(0.02)^{x}}{x!}$$

- First, we need the value of $e^{-\lambda} = e^{-0.02}$.
  - (Using the table/appendix as referenced in the source)

$$e^{-0.02} \approx 0.9802$$

Step 3: Calculate Packets with One Defective ($X = 1$)

- **Probability $P[X = 1]$:**

$$P[X = 1] = \frac{e^{-0.02}(0.02)^1}{1!} \approx 0.9802 \times 0.02$$

Step 3: Calculate Packets with One Defective ($X = 1$)

- **Probability $P[X = 1]$:**

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- **Expected Frequency $f(1)$:**

$$f(1) = N \cdot P[X = 1] = 20000 \times P[X = 1]$$

- **Calculation:**

$$f(1) = 20000 \times (0.9802) \times (0.02)$$
$$f(1) = 20000 \times 0.019604$$

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- **Probability $P[X = 1]$:**

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- **Expected Frequency $f(1)$:**

$$f(1) = N \cdot P[X = 1] = 20000 \times P[X = 1]$$

- **Calculation:**

$$f(1) = 20000 \times (0.9802) \times (0.02)$$
$$f(1) = 20000 \times 0.019604$$

- **Result:**

$$f(1) \approx 392.08$$

The approximate number of packets is 392.

Step 4: Calculate Packets with Two Defectives ($X = 2$)

- **Probability $P[X = 2]$:**

$$P[X = 2] = \frac{e^{-0.02}(0.02)^2}{2!} \approx \frac{0.9802 \times (0.0004)}{2}$$

Step 4: Calculate Packets with Two Defectives ($X = 2$)

- **Probability $P[X = 2]$:**

$$P[X = 2] = \frac{e^{-0.02}(0.02)^2}{2!} \approx \frac{0.9802 \times (0.0004)}{2}$$

- **Expected Frequency $f(2)$:**

$$f(2) = N \cdot P[X = 2] = 20000 \times P[X = 2]$$

Step 4: Calculate Packets with Two Defectives ($X = 2$)

- **Probability $P[X = 2]$:**

$$P[X = 2] = \frac{e^{-0.02}(0.02)^2}{2!} \approx \frac{0.9802 \times (0.0004)}{2}$$

- **Expected Frequency $f(2)$:**

$$f(2) = N \cdot P[X = 2] = 20000 \times P[X = 2]$$

- **Calculation:**

$$f(2) = 20000 \times \frac{0.9802 \times 0.0004}{2}$$
$$f(2) = 20000 \times \frac{0.00039208}{2}$$
$$f(2) = 20000 \times 0.00019604$$

Step 4: Calculate Packets with Two Defectives ($X = 2$)

- **Probability $P[X = 2]$:**

$$P[X = 2] = \frac{e^{-0.02}(0.02)^2}{2!} \approx \frac{0.9802 \times (0.0004)}{2}$$

- **Expected Frequency $f(2)$:**

$$f(2) = N \cdot P[X = 2] = 20000 \times P[X = 2]$$

- **Calculation:**

$$f(2) = 20000 \times \frac{0.9802 \times 0.0004}{2}$$
$$f(2) = 20000 \times \frac{0.00039208}{2}$$
$$f(2) = 20000 \times 0.00019604$$

- **Result:**

$$f(2) \approx 3.9208$$

The approximate number of packets is 4.

Summary of Results

- **Parameters:** $\lambda = 0.02$, $N = 20000$.
- **Part (i): One defective pen** $(X = 1)$
  - $f(1) \approx 392.08$
  - **Approximate Number of Packets: 392**

Summary of Results

- Parameters: $\lambda = 0.02$, $N = 20000$.
- Part (i): One defective pen ($X = 1$)
  - $f(1) \approx 392.08$
  - Approximate Number of Packets: 392
- Part (ii): Two defective pens ($X = 2$)
  - $f(2) \approx 3.9208$
  - Approximate Number of Packets: 4

Example: Problem Statement

Data on Mistakes per Page

A typist committed the following mistakes per page in typing 100 pages. The goal is to fit a Poisson distribution and calculate the theoretical frequencies.

Mistakes per page (X)	0	1	2	3	4	5
Frequency (f)	42	33	14	6	4	1

Step 1: Calculate the Mean (λ)

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- **Result:**

$$\lambda = 1$$

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- **Result:**

$$p(0) = 0.3679$$

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  - $p(5)$: $p(4) \times \frac{1}{5} \approx 0.0153 \times 0.2 \approx 0.0031$

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  - $f(5)$: $100 \times 0.0031 \approx 0.31 \rightarrow 0$

Conclusion: Observed vs. Theoretical Frequencies

Summary of Results

The Poisson distribution with a mean of $\lambda = 1$ is fitted to the data. The theoretical frequencies show a close match to the observed frequencies.

Mistakes (X)	0	1	2	3	4	5	Total
Observed (f)	42	33	14	6	4	1	100
Theoretical ($f(x)$)	37	37	18	6	2	0	100

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Thank You!