Discrete Uniform Distribution

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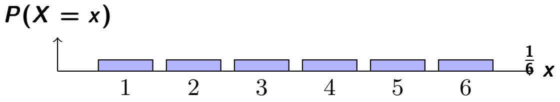
- Example: If X is the outcome of rolling an unbiased die, then $a = 1, b = 6 \implies n = 6$.
- Hence,

$$P(X = x) = \frac{1}{6}, \quad x = 1, 2, 3, 4, 5, 6.$$

Visualization: Uniform Distribution for a Die Roll



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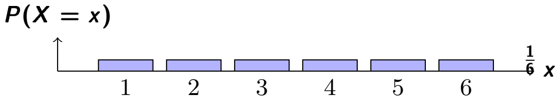

Observation

All outcomes $\mathbf{1, 2, 3, 4, 5, 6}$ are equally likely, each with probability $\frac{1}{6}$.

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- Simplifying, we get

$$E[X] = \frac{1}{n} \sum_{x=a}^{b} x = \frac{a+b}{2}.$$

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$$E[X^2] = \frac{1}{n}\sum_{x=a}^{b} x^2.$$

- Using the formula for sum of squares,

$$E[X^2] = \frac{(b-a+1)(a^2+ab+b^2)}{3n}.$$

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- Hence,

$$Var(X) = \frac{(n^2-1)}{12}.$$

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- Mean: $E[X] = \frac{n+1}{2} = \frac{7}{2}$.

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Solution:

- Let X be the number on an unbiased die. Then $n = 6$.
- Values: $1, 2, 3, 4, 5, 6$.
- $P(X = x) = \frac{1}{6}$.
- Mean: $E[X] = \frac{n+1}{2} = \frac{7}{2}$.
- Variance: $Var(X) = \frac{n^2-1}{12} = \frac{35}{12}$.

Expected Frequency of Die Outcomes

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If an unbiased die is thrown 120 times, find the expected frequency of 1–6.

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If an unbiased die is thrown 120 times, find the expected frequency of 1–6.

- $P(X = x) = \frac{1}{6}$.
- $f(x) = N \cdot P(X = x) = 120 \times \frac{1}{6} = 20$.
- Hence, each face is expected to appear 20 times.

Expected Frequency Table

Expected Frequency Table

X	$P(X = x)$	Expected Frequency $f(x)$
1	$\frac{1}{6}$	20
2	$\frac{1}{6}$	20
3	$\frac{1}{6}$	20
4	$\frac{1}{6}$	20
5	$\frac{1}{6}$	20
6	$\frac{1}{6}$	20

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Discrete Uniform Random Variable Expected Frequencies

- X is the number on a ticket from 1 to 10.
- $P(X = x) = \frac{1}{10}, \; x = 1, 2, \ldots, 10.$
- $N = 150.$
- $f(x) = N \cdot P(X = x) = 150 \cdot \frac{1}{10} = 15.$

Expected Frequencies Table

X	$P(X = x)$	Expected/Theoretical frequency $f(x) = N \cdot P[X = x]$ $= 150 \cdot P[X = x]$
1	$\frac{1}{10}$	$150 \times \frac{1}{10} = 15$
2	$\frac{1}{10}$	$150 \times \frac{1}{10} = 15$
3	$\frac{1}{10}$	$150 \times \frac{1}{10} = 15$
4	$\frac{1}{10}$	$150 \times \frac{1}{10} = 15$
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6	$\frac{1}{10}$	$150 \times \frac{1}{10} = 15$
7	$\frac{1}{10}$	$150 \times \frac{1}{10} = 15$
8	$\frac{1}{10}$	$150 \times \frac{1}{10} = 15$
9	$\frac{1}{10}$	$150 \times \frac{1}{10} = 15$
10	$\frac{1}{10}$	$150 \times \frac{1}{10} = 15$

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