Normal Distribution

A Detailed Step-by-Step Explanation

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- In this section, we focus on the **Continuous Random Variable** and the most important one: the **Normal Distribution**.

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- The most common continuous distribution is the **Normal Distribution**.

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- The curve is symmetric about the mean μ.
- The spread (width) depends on the standard deviation σ.

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- This is the most commonly used form for normal probability tables.

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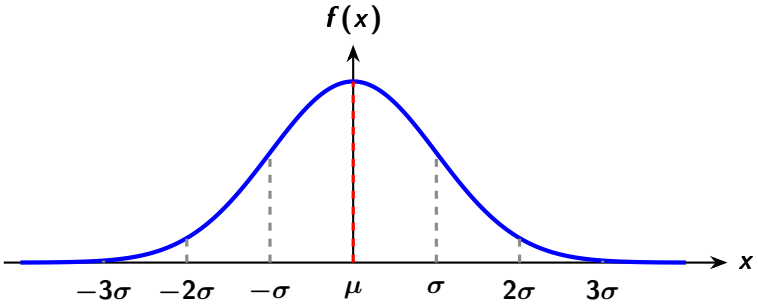
- **Empirical Rule:**

$$\begin{aligned} \text{Within } \mathbf{1}\sigma &\Rightarrow \mathbf{68.27\%} \\ \text{Within } \mathbf{2}\sigma &\Rightarrow \mathbf{95.45\%} \\ \text{Within } \mathbf{3}\sigma &\Rightarrow \mathbf{99.73\%} \end{aligned}$$

Graph of Normal Distribution



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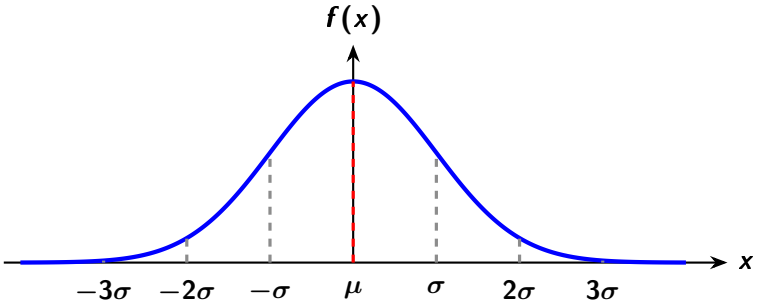

- The gray lines mark positions of standard deviations ($\boldsymbol{\sigma}$) from the mean.
- About 68%, 95%, and 99.7% of data lie within $1\boldsymbol{\sigma}$, $2\boldsymbol{\sigma}$, and $3\boldsymbol{\sigma}$ respectively.

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- The Central Limit Theorem states that the sum of many independent random variables tends to a normal distribution.

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