Support Vector Machine (SVM)

A Simple Numerical Example Detailed Explanation

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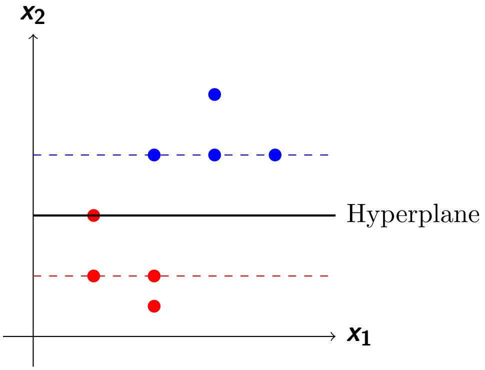
- Find the **optimal hyperplane** that separates classes
- **Maximize margin** between support vectors and boundary
- **Handle non-linear data** using kernel trick



Dataset Setup: Table of 8 Points

- Two classes: $\mathbf{+1}$ (Blue), $\mathbf{-1}$ (Red)

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- Two classes: $+1$ (Blue), -1 (Red)
- Linearly separable by the hyperplane $x_2 = 2$

Point	Coordinates (x_1, x_2)	Class y_i
P_1	$(2, 3)$	$+1$
P_2	$(3, 3)$	$+1$
P_3	$(3, 4)$	$+1$
P_4	$(4, 3)$	$+1$
P_5	$(1, 1)$	-1
P_6	$(2, 1)$	-1
P_7	$(1, 2)$	-1
P_8	$(2, 0.5)$	-1

Step 1: Propose a Separating Hyperplane

- Suppose $\boldsymbol{w} = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix}, \boldsymbol{b} = -\mathbf{2}$

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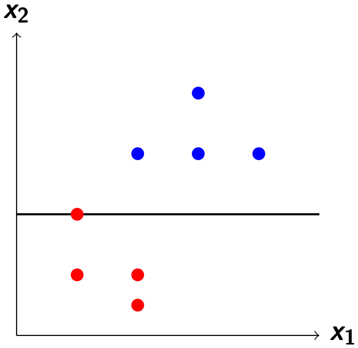
- Suppose $\boldsymbol{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \boldsymbol{b} = -2$
- Decision boundary equation: $\boldsymbol{w}^T \boldsymbol{x} + \boldsymbol{b} = 0 \Rightarrow \boldsymbol{x}_2 = 2$



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- Check all points:
  - Class $+1$: $(2,3) \to 1$, $(3,3) \to 1$, $(3,4) \to 2$, $(4,3) \to 1$
  - Class -1: $(1,1) \to -1$, $(2,1) \to -1$, $(1,2) \to 0$, $(2,0.5) \to -1.5$

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- All points correctly classified except $(1,2)$ exactly on the hyperplane

SVM Constraint: $y_i(w^T x_i + b) \geq 1$

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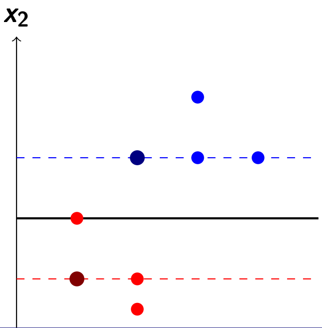
- Points on the margin satisfy $y_i(w^T x_i + b) = 1 \rightarrow$ **support vectors**



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- Distance from point to hyperplane:

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- Margin width $= d_+ + d_- = 2$

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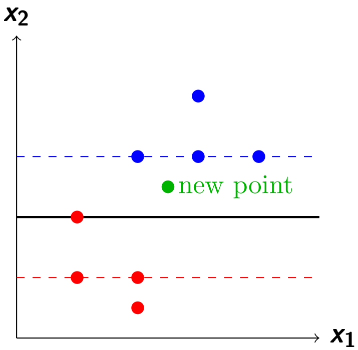
- Distance to hyperplane $= 0.5$



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- Separating hyperplane: $\boldsymbol{x_2 = 2}$
- Weight vector: $\boldsymbol{w = [0, 1]^T}$, bias $\boldsymbol{b = -2}$
- Margin width $= 2$, support vectors highlighted
- SVM constraint ensures all points outside margin
- New point $\mathbf{(2.5, 2.5)}$ correctly classified as $\mathbf{+1}$

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